CABLES

Instructional Objectives:
After reading this chapter the student will be able to
1. Differentiate between rigid and deformable structures.
2. Define funicular structure.
3. State the type stress in a cable.
4. Analyse cables subjected to uniformly distributed load.
5. Analyse cables subjected to concentrated loads.

31.1 Introduction
Cables and arches are closely related to each other and hence they are grouped in this course in the same module. For long span structures (for e.g. in case bridges) engineers commonly use cable or arch construction due to their efficiency. In the first lesson of this module, cables subjected to uniform and concentrated loads are discussed. In the second lesson, arches in general and three hinged arches in particular along with illustrative examples are explained. In the last two lessons of this module, two hinged arch and hingeless arches are considered.
Structure may be classified into rigid and deformable structures depending on change in geometry of the structure while supporting the load. Rigid structures support externally applied loads without appreciable change in their shape (geometry). Beams trusses and frames are examples of rigid structures. Unlike rigid structures, deformable structures undergo changes in their shape according to externally applied loads. However, it should be noted that deformations are still small. Cables and fabric structures are deformable structures. Cables are mainly used to support suspension roofs, bridges and cable car system. They are also used in electrical transmission lines and for structures supporting radio antennas. In the following sections, cables subjected to concentrated load and cables subjected to uniform loads are considered.

The shape assumed by a rope or a chain (with no stiffness) under the action of external loads when hung from two supports is known as a funicular shape. Cable is a funicular structure. It is easy to visualize that a cable hung from two supports subjected to external load must be in tension (vide Fig. 31.2a and 31.2b). Now let us modify our definition of cable. A cable may be defined as the structure in pure tension having the funicular shape of the load.

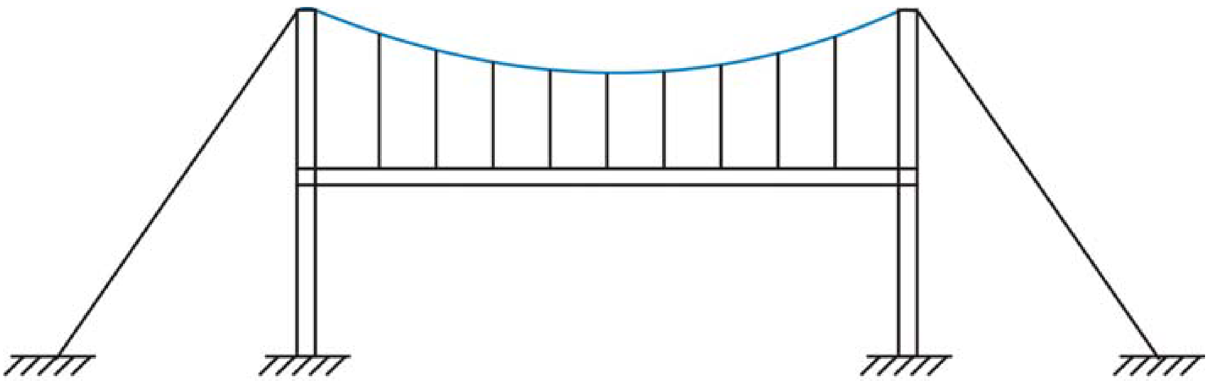

Fig. 31.1 Deformable structure.

Fig 31.2a Unloaded cable (when dead load is neglected)

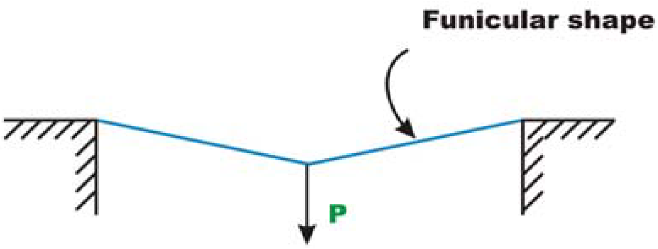

Figure 31.2b Cable in tension.

31.2 Cable subjected to Concentrated Loads

As stated earlier, the cables are considered to be perfectly flexible (no flexural stiffness) and inextensible. As they are flexible they do not resist shear force and bending moment. It is subjected to axial tension only and it is always acting tangential to the cable at any point along the length. If the weight of the cable is negligible as compared with the externally applied loads then its self weight is neglected in the analysis. In the present analysis self weight is not considered.

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Consider a cable $ACDEB$ as loaded in Fig. 31.2. Let us assume that the cable lengths L_1, L_2, L_3, L_4 and sag at C, D, E (h_c, h_d, h_e) are known. The four reaction components at A and B, cable tensions in each of the four segments and three sag values: a total of eleven unknown quantities are to be determined. From the geometry, one could write two force equilibrium equations ($\sum F_x = 0, \sum F_y = 0$) at each of the point A, B, C, D and E i.e. a total of ten equations and the required one more equation may be written from the geometry of the cable. For example, if one of the sag is given then the problem can be solved easily. Otherwise if the total length of the cable S is given then the required equation may be written as

$$S = \sqrt{L_1^2 + h_c^2} + \sqrt{L_2^2 + (h_d - h_c)^2} + \sqrt{L_2^2 + (h_d - h_e)^2} + \sqrt{L_2^2 + (h + h_e)^2} \qquad (31.1)$$

31.3 Cable subjected to uniform load.

Cables are used to support the dead weight and live loads of the bridge decks having long spans. The bridge decks are suspended from the cable using the hangers. The stiffened deck prevents the supporting cable from changing its shape by distributing the live load moving over it, for a longer length of cable. In such cases cable is assumed to be uniformly loaded.

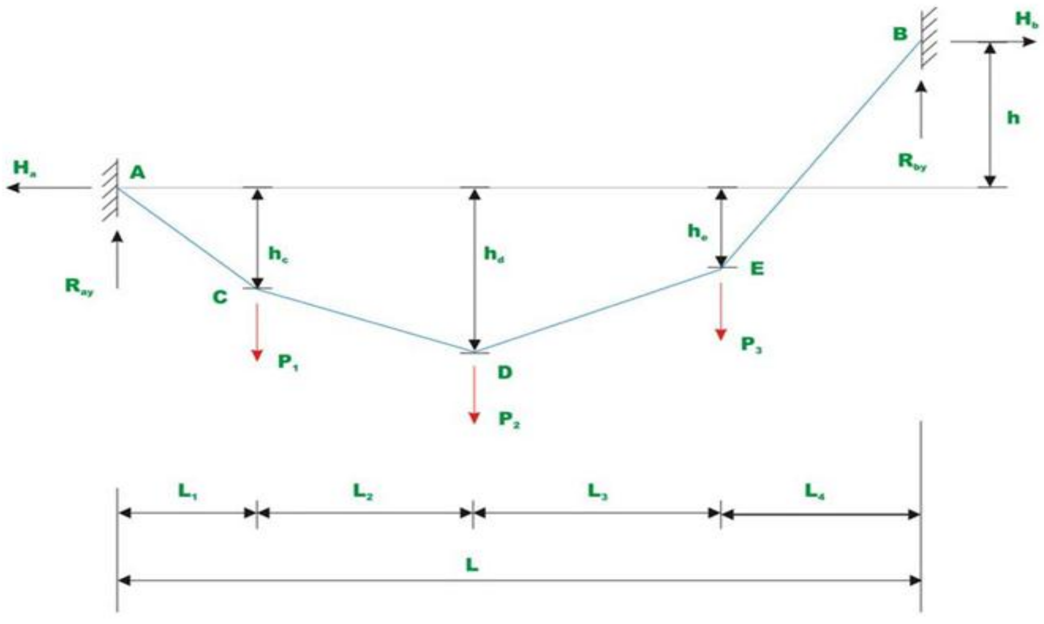
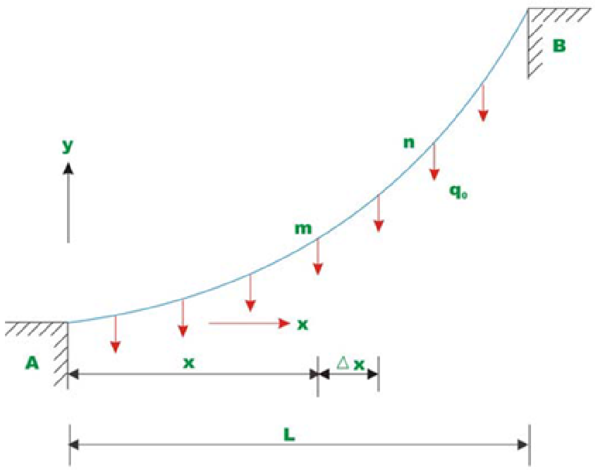


Fig. 31.3a Cable subjected to concentrated load.



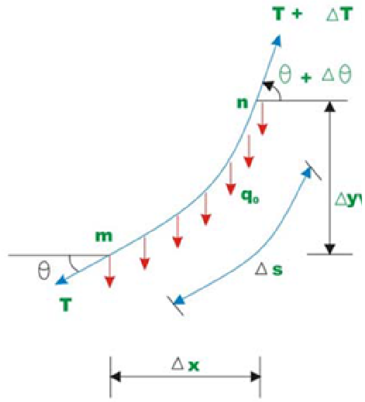


Fig. 31.3b Cable subjected to uniformly distributed load. **Fig. 31.3c Free-body diagram**

Consider a cable which is uniformly loaded as shown in Fig 31.3a. Let the slope of the cable be zero at A. Let us determine the shape of the cable subjected to uniformly distributed load q_0 .Consider a free body diagram of the cable as shown in Fig 31.3b. As the cable is uniformly loaded, the tension in the cable changes continuously along the cable length. Let the tension in the cable at m end of the free body diagram be T and tension at the n end of the cable be $T + \Delta T$. The slopes of the cable at m and n are denoted by θ and $\theta + \Delta\theta$ respectively. Applying equations of equilibrium, we get

$$\sum Fy = 0 \qquad -T\sin\theta + (T+\Delta T)\sin(\theta+\Delta\theta) - q_0(\Delta x) = 0 \qquad (31.2a)$$

$$\sum Fx = 0 \qquad -T\cos\theta + (T+\Delta T)\cos(\theta+\Delta\theta) = 0 \qquad (31.2b)$$

$$\sum Mn = 0 \qquad -(T\cos\theta)\Delta y + (T\sin\theta)\Delta x + (q_0\Delta x)\frac{\Delta x}{2} = 0 \qquad (31.2c)$$

Dividing equations 31.2a, b, c by Δx and noting that in the limit as $\Delta x \to 0, \Delta y \to 0$ $\Delta\theta \to 0$ and $\Delta T \to 0$.

$$\lim_{\Delta x \to 0} \quad \frac{\Delta T}{\Delta x}\sin(\theta+\Delta\theta) = q_0$$

$$\frac{d}{dx}(T\sin\theta) = q_0 \qquad (31.3a)$$

$$\frac{d}{dx}(T\cos\theta) = 0 \qquad (31.3b)$$

$$\lim_{\Delta x \to 0} \quad -T\cos\theta\frac{\Delta y}{\Delta x} + T\sin\theta + q_0\frac{x_0}{2} = 0$$

$$\frac{dy}{dx} = \tan\theta \tag{31.3c}$$

Integrating equation (31.3b) we get

$$T\cos\theta = \text{constant}$$

At support (i.e., at $x = 0$), $\qquad T\cos\theta = H \qquad$ (31.4a)

i.e. horizontal component of the force along the length of the cable is constant.

Integrating equation 31.3a,

$$T\sin\theta = q_0 x + C_1$$

At $x = 0$, $\qquad T\sin\theta = 0$, $\qquad C_1 = 0 \qquad$ as $\theta = 0$ at that point.

Hence, $\qquad T\sin\theta = q_0 x \qquad$ (31.4b)

From equations 31.4a and 31.4b, one could write

$$\tan\theta = \frac{q_0 x}{H} \tag{31.4c}$$

From the figure, $\qquad \tan\theta = \frac{dy}{dx} = \frac{q_0 x}{H}$

$$\therefore y = \frac{q_0 x^2}{2H} + C$$

At $x = 0, y = 0 \Rightarrow C = 0$ and $y = \dfrac{q_0 x^2}{2H} \qquad$ (31.5)

Equation 31.5 represents a parabola. Now the tension in the cable may be evaluated from equations 31.4a and 31.4b. i.e,

$$T = \sqrt{q_0^{\,2} x^2 + H^2}$$

$T = T_{\max}$, $\qquad$ when $x = L$.

$$T_{\max} = \sqrt{q_0^{\,2} L^2 + H^2} = q_0 L \sqrt{1 + \left(\frac{H}{q_0 L}\right)^2} \tag{31.6}$$

Due to uniformly distributed load, the cable takes a parabolic shape. However due to its own dead weight it takes a shape of a catenary. However dead weight of the cable is neglected in the present analysis.

Example 31.1

Determine reaction components at A and B, tension in the cable and the sag y_B, and y_E of the cable shown in Fig. 31.4a. Neglect the self weight of the cable in the analysis.

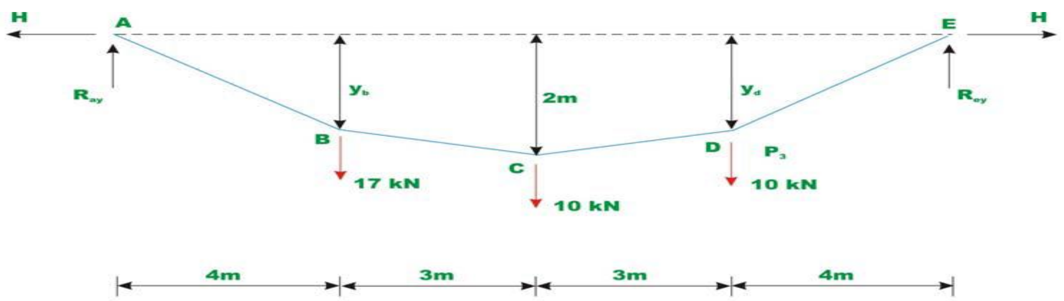
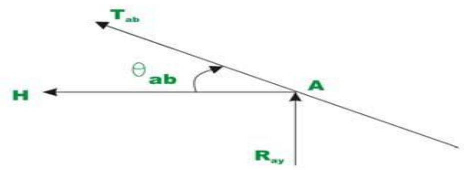
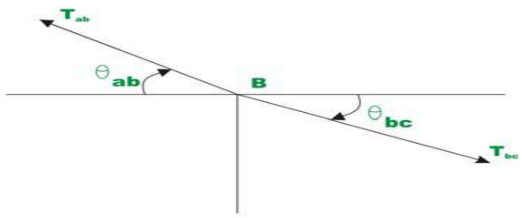


Fig. 31.4 Example 31.1



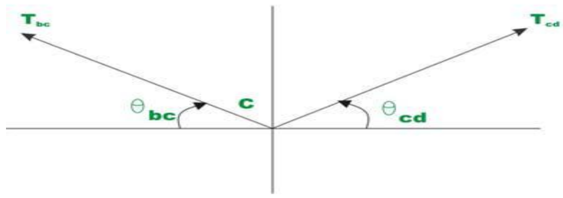
Joint A

Fig. 31.4b



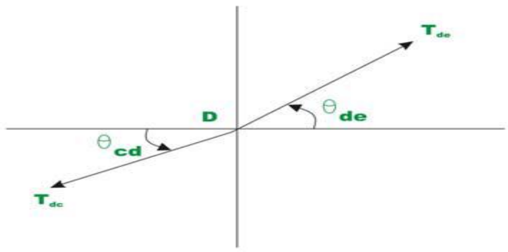
Joint B

Fig. 31.4c



Joint C

Fig. 31.4d



Joint D

Fig. 31.4e

Since there are no horizontal loads, horizontal reactions at A and B should be the same. Taking moment about *E*, yields

$$R_{ay} \times 14 - 17 \times 20 - 10 \times 7 - 10 \times 4 = 0$$

$$R_{ay} = \frac{280}{14} = 20 \quad \text{kN}; \qquad R_{ey} = 37 - 20 = 17\text{kN}.$$

Now horizontal reaction H may be evaluated taking moment about point C of all forces left of C.

$$R_{ay} \times 7 - H \times 2 - 17 \times 3 = 0$$

$$H = 44.5 \text{ kN}$$

Taking moment about B of all the forces left of B and setting $M_B = 0$, we get

$$R_{ay} \times 4 - H \times y_B = 0; \qquad y_B = \frac{80}{44.50} = 1.798\,m$$

Similarly, $y_D = \dfrac{68}{44.50} = 1.528\,m$

To determine the tension in the cable in the segment AB, consider the equilibrium of joint A (vide Fig.31.4b).

$$\sum F_x = 0 \Rightarrow T_{ab} \cos\theta_{ab} = H$$

$$T_{ab} = \frac{44.5}{\left(3 \Big/ \sqrt{3^2 + 0.298^2} \right)} = 48.789 \text{ kN}$$

The tension T_{ab} may also be obtained as

$$T_{ab} = \sqrt{R_{ay}{}^2 + H^2} = \sqrt{20^2 + 44.5^2} = 48.789 \quad \text{kN}$$

Now considering equilibrium of joint $B, C,$ and D one could calculate tension in different segments of the cable.

Segment bc

Applying equations of equilibrium,

$$\sum F_x = 0 \Rightarrow T_{ab} \cos\theta_{ab} = T_{bc} \cos\theta_{bc}$$

$$T_{bc} = \frac{44.5}{\left(3 \Big/ \sqrt{3^2 + 0.298^2} \right)} \cong 44.6 \quad \text{kN}$$

See Fig.31.4c

Segment cd

$$T_{cd} = \frac{T_{bc} \cos\theta_{bc}}{\cos\theta_{cd}} = \frac{44.5}{\left(3 \Big/ \sqrt{3^2 + 0.472^2} \right)} = 45.05 \quad \text{kN}$$

See Fig.31.4d.
See Fig.31.4e.

Segment de

$$T_{de} = \frac{T_{cd} \cos\theta_{cd}}{\cos\theta_{de}} = \frac{44.5}{4 \Big/ \sqrt{4^2 + 1.528^2}} = 47.636 \quad \text{kN}$$

The tension T_{de} may also be obtained as

$$T_{de} = \sqrt{R_{ey}{}^2 + H^2} = \sqrt{17^2 + 44.5^2} = 47.636 \quad \text{kN}$$

Example 31.2

A cable of uniform cross section is used to span a distance of 40m as shown in Fig 31.5. The cable is subjected to uniformly distributed load of 10 kN/m. run. The left support is below the right support by 2 m and the lowest point on the Cable C is located below left support by 1 m. Evaluate the reactions and the maximum and minimum values of tension in the cable.

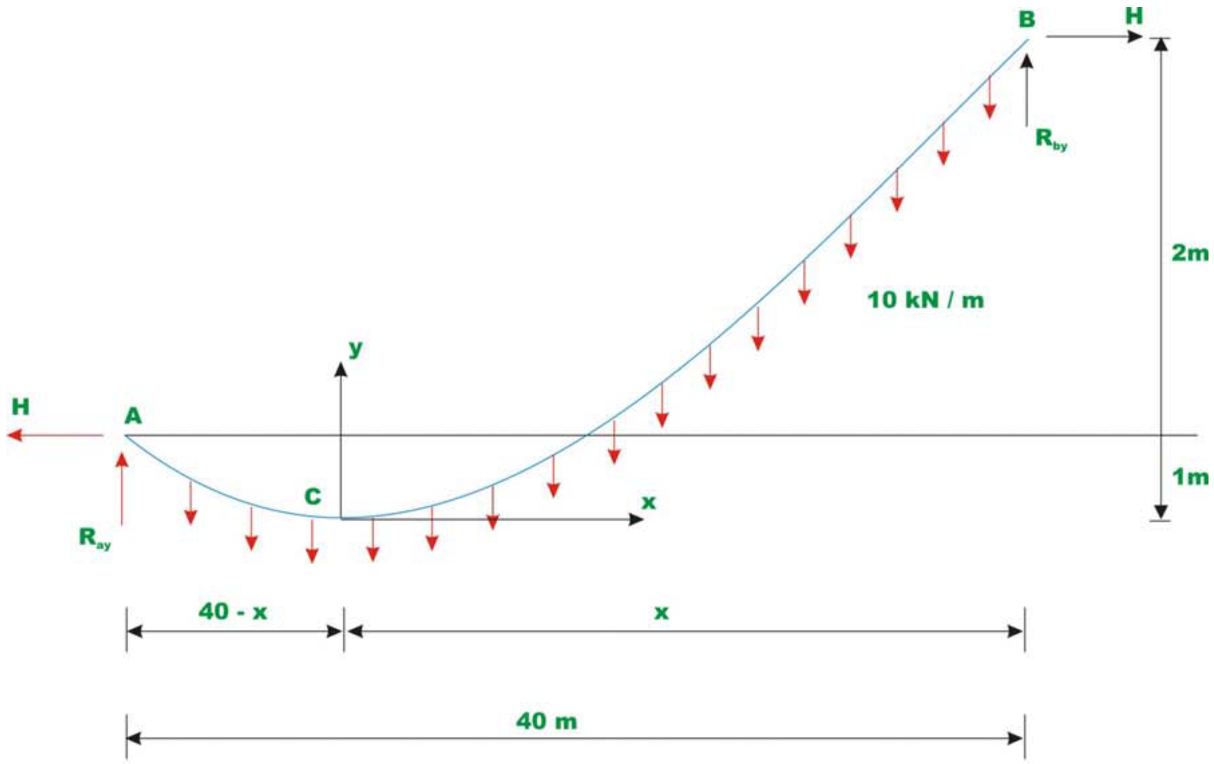


Fig. 31.5 Example 31.2

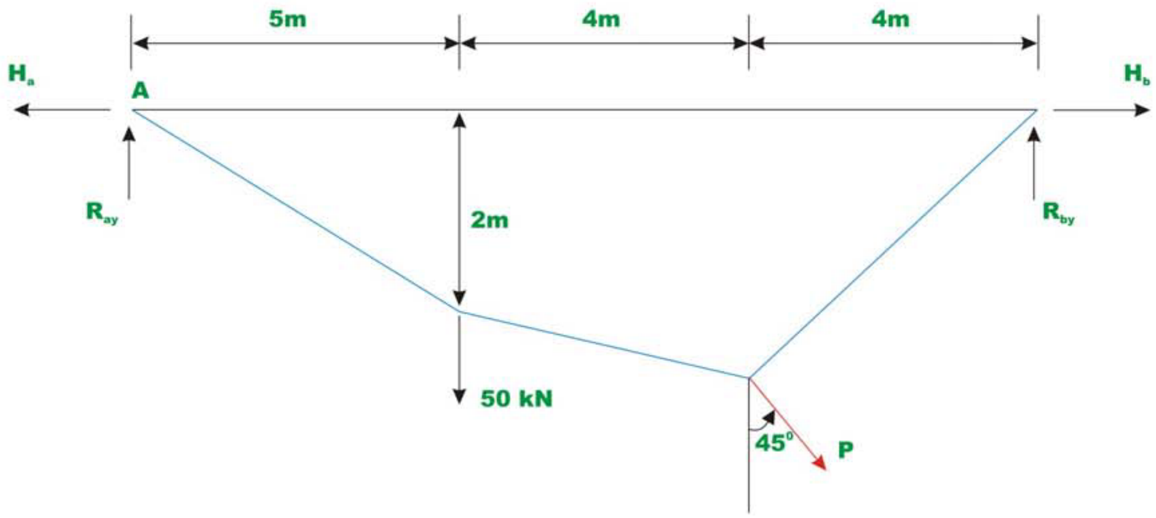


Fig. 31.6 Example 31.3

Assume the lowest point C to be at distance of x m from B. Let us place our origin of the co-ordinate system xy at C. Using equation 31.5, one could write,

$$y_a = 1 = \frac{q_0(40-x)^2}{2H} = \frac{10(40-x)^2}{2H} \qquad (1)$$

$$y_b = 3 = \frac{10x^2}{2H} \qquad (2)$$

where y_a and y_b be the y co-ordinates of supports A and B respectively. From equations 1 and 2, one could evaluate the value of x. Thus,

$$10(40-x)^2 = \frac{10x^2}{3} \quad \Rightarrow \quad x = 25.359\,m$$

From equation 2, the horizontal reaction can be determined.

$$H = \frac{10 \times 25.359^2}{6} = 1071.80 \ \ \text{kN}$$

Now taking moment about A of all the forces acting on the cable, yields

$$R_{by} = \frac{10 \times 40 \times 20 + 1071.80 \times 2}{40} = 253.59 \ \ \text{kN}$$

Writing equation of moment equilibrium at point B, yields

$$R_{ay} = \frac{40 \times 20 \times 10 - 1071.80 \times 2}{40} = 146.41 \ \ \text{kN}$$

Tension in the cable at supports A and B are

$$T_A = \sqrt{146.41^2 + 1071.81^2} = 1081.76 \ \ \text{kN}$$

$$T_B = \sqrt{253.59^2 + 1071.81^2} = 1101.40 \ \ \text{kN}$$

The tension in the cable is maximum where the slope is maximum as $T\cos\theta = H$. The maximum cable tension occurs at B and the minimum cable tension occurs at C where $\frac{dy}{dx} = \theta = 0$ and $T_C = H = 1071.81 \ \ \text{kN}$

Example 31.3

A cable of uniform cross section is used to support the loading shown in Fig 31.6. Determine the reactions at two supports and the unknown sag c. y

Taking moment of all the forces about support B,

$$R_{ay} = \frac{1}{10}[350 + 300 + 100 y_c] \qquad (1)$$

$$R_{ay} = 65 + 10 y_c$$

Taking moment about B of all the forces left of B and setting $M_B = 0$, we get,

$$R_{ay} \times 3 - H_a \times 2 = 0$$
$$\Rightarrow H_a = 1.5 R_{ay} \qquad (2)$$

Taking moment about C of all the forces left of C and setting $M_C = 0$, we get

$$\sum M_C = 0 \qquad R_{ay} \times 7 - H_a \times y_C - 50 \times 4 = 0$$

Substituting the value of H_a in terms of R_{ay} in the above equation,

$$7R_{ay} - 1.5 R_{ay} y_C - 200 = 0 \qquad (3)$$

Using equation (1), the above equation may be written as,

$$y_C^2 + 1.833 y_C - 17 = 0 \qquad (4)$$

Solving the above quadratic equation, y_C can be evaluated. Hence,

$y_C = 3.307m$.

Substituting the value of y_C in equation (1),

$R_{ay} = 98.07$ kN

From equation (2),

$H_a = 1.5 R_{ay} = 147.05$ kN

Now the vertical reaction at D, R_{dy} is calculated by taking moment of all the forces about A,

$$R_{dy} \times 10 - 100 \times 7 + 100 \times 3.307 - 50 \times 3 = 0$$

$$R_{dy} = 51.93 \text{ kN.}$$

Taking moment of all the forces right of C about C, and noting that $\sum M_C = 0$,

$$R_{dy} \times 3 = H_d \times y_C \quad \Rightarrow H_d = 47.109 \text{ kN.}$$

Summary

In this lesson, the cable is defined as the structure in pure tension having the funicular shape of the load. The procedures to analyse cables carrying concentrated load and uniformly distributed loads are developed. A few numerical examples are solved to show the application of these methods to actual problems.